

EDM Collaboration,  
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# EDM Systematics, as they follow from spin dynamics

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# Outline

1. Method
2.  $E_V$
3.  $B_R$  without  $E_V$
4. Correlation  $\langle B_L(s) \delta\omega_a(s) \rangle$
5. Correlation  $\langle B_R(s) \delta\omega_a(s) \rangle$
6. Pitch effect
7.  $\Delta p/p$  : dephasing
8.  $\Delta p/p$  : correlation  $\langle B_L(s) \Omega_a(s) \rangle$ ,  
 $\Omega_a \equiv d\delta\omega_a/dp$
9. Tolerances for  $\mu, d$ .

# method

1. BMT
2. Covariant BMT in cylindrical coordinates
3. Why?
4. A "dynamic" rest-frame for spin.

3a

## BMT

$$S = (S^1, S^2, S^3, S^0) \rightarrow (\vec{S}, 0)$$

$$\vec{S} = (S_L, S_R, S_V); \quad \vec{S}^2 = 1.$$

$$S^k S_k \equiv S^0 S_0 + S^1 S_1 + S^2 S_2 + S^3 S_3 = -1$$

$$p = mu = (p^1, p^2, p^3, p^0) \rightarrow (0, m)$$

$$S u \equiv S^1 u_1 + S^2 u_2 + S^3 u_3 + S^0 S_0 = 0,$$

because in the rest frame  $S = (\vec{S}, 0), p = (0, m)$

Eq's:

$$\frac{dS^i}{dt} = \frac{e}{m} \left[ \frac{g}{2} F_{ik}^i S^k + \frac{g-2}{2} u^i (F_{ke} S^k u^e) \right]$$

In the cylindrical coordinates:

$d \rightarrow D$  (to include the analogue of centrifugal forces)

$$\frac{D S^i}{dt} = \frac{e}{m} \left[ \frac{g}{2} F_{ik}^i S^k + \frac{g-2}{2} u^i (F_{ke} S^k u^e) \right] +$$

(g-2) Note No. 279

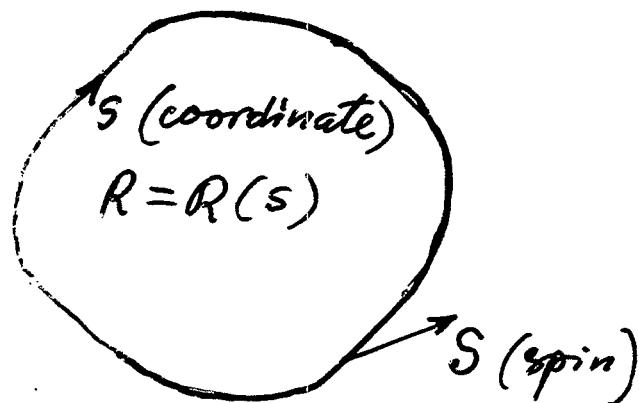
$$+ \frac{q}{2m} e E^{ikl} \tilde{F}_{kq} u^q S_e^l u_j \quad (\text{EDM})$$

YEDM Note No. 10)

3b

## BMT

Why cylindrical?



$$S_L = 1 \text{ when } \vec{s} \parallel \vec{p}$$

$\Omega_s = 0$  when this is not changed in time,

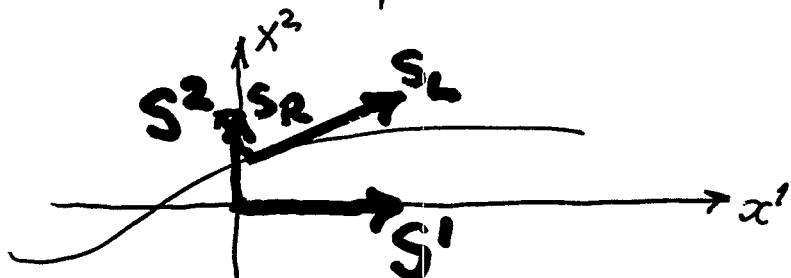
The resulting eq's are,  
on one hand, such that  
as if we use the (wrong)  
formula  $\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_p$ , and,  
on the other hand, are  
fundamentally precise.

## BMT

In any concrete problem, only some limited types of field and trajectories are involved → the final eq's are not so complicated.

After that → to the rest frame for spin (not for fields) → a "cylindrical" analogy of the Thomas-BMT.

Example



$$S^1 = \gamma S_L - \frac{u^2}{u'} S_R + O\left(\frac{u^2}{u'}\right)^2$$

$$S^2 = -\gamma \frac{u^2}{u'} S_L + S_R + O\left(\frac{u^2}{u'}\right)^2$$

$$S^0 = u' S_L$$

# $E_v$

1. Why only  $E_v$ ?  
What about  $B_R$ ?
2. Fields at the perturbed orbits.
3. Spin eq's, taking into account beam eq's in the case of  $E_v, B_R$ .
4. Tolerances for  $\mu, d$ .

4a

 $E_v$ 

In our eq's (as in Thomas-BMT's),  
 the fields are defined at the actual  
 (perturbed) trajectories.

→ Look at trajectories

$$\frac{du^3}{dt} = \frac{e}{m} \left( F_0^3 u^0 + F^3, u^1 + F^3, \frac{1}{2} u^2 \right)$$

If there are only  $E_v$  and lenses  
 If we investigate only the  
 average along the orbit

$$F_0^3 u^0 + F^3, u^1 = 0$$

$$\frac{dS^3}{dt} = \frac{e}{m} \left[ (1+\alpha) \left( F^3, S^1 + F_0^3 S^0 \right) + \frac{\gamma}{2} (E_R - \beta B_v) S^1 \right]$$

$$S^0 = \frac{u^1}{u^0} S^1 \quad (\text{in this case})$$

$$\Rightarrow \frac{dS_v}{dt} = \frac{e}{m} \left[ (1+\alpha) \frac{E_v}{\beta j^2} + \frac{\gamma}{2} (E_R - \beta B_v) \right] S_L$$

$$F_{02} \equiv E_v \ll \frac{\gamma \beta^2}{2(1+\alpha)} \gamma^2 B_v$$

EDM Note N

Will be in EDM Note No. 34  
(The draft to be corrected)

$B_R$  without  $E_v$

1. Beam physics :

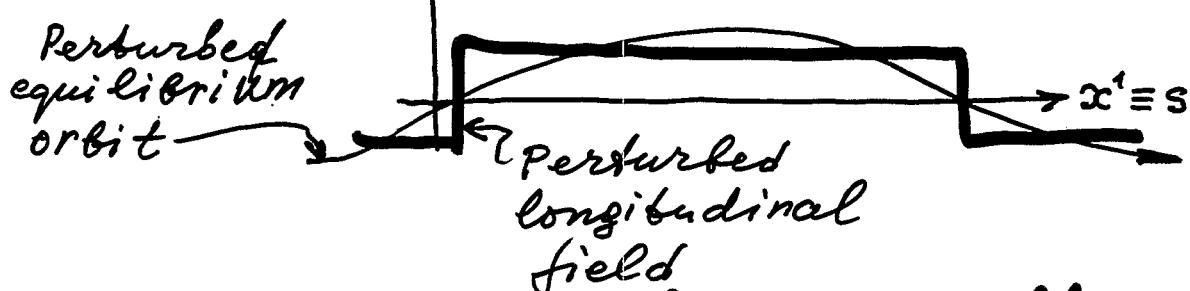
$$B_R = \langle B_L \theta_R \rangle$$

2. Spin physics :

Will be shown

→ NO  $B_R$  in the rest frame for S and the fields.

$$\frac{dP_v}{dt} = e [v_L B_R - v_R B_L] \rightarrow 0 \quad \text{at the equilibrium.}$$



If  $\langle \theta_R(s) B_L(s) \rangle \neq 0$ , then

$$\langle B_R \rangle = \frac{1}{v_L} \langle \theta_R(s) B_L(s) \rangle \neq 0$$

5a

$B_R$  without  $E_V$

$$\frac{dS^3}{dt} = \frac{e}{m}(1+\alpha)[F^3, S' + F_2^3 S^2] + \frac{e}{m} \frac{1}{2} (E_R - \beta B_V) S'$$

$$F_{11}^3 = F_{13} = B_R = \langle \theta_R B_L \rangle$$

$$F_{23}^3 = F_{23} = B_L$$

$$S' = \gamma S_L + \dots \text{ (linear appr. in)}$$

$$S^3 = S_V$$

$$S^2 = S_R - \gamma \theta_R S_L$$

p. 3c

$$F^3, S' + F_2^3 S^2 = \underbrace{\langle \theta_R B_L \rangle}_{\gamma S_L} +$$

$$+ B_L S_R - \underbrace{\gamma \langle \theta_R B_L \rangle S_L}$$

$$\gamma \tau = t$$

$$\frac{dS_V}{dt} = \frac{e}{m}(1+\alpha) B_L(s) S_R + \frac{e}{m} \frac{1}{2} (E_R - \beta B_V)$$

$\langle B_L(s) \rangle = 0$  NO EFFECT!

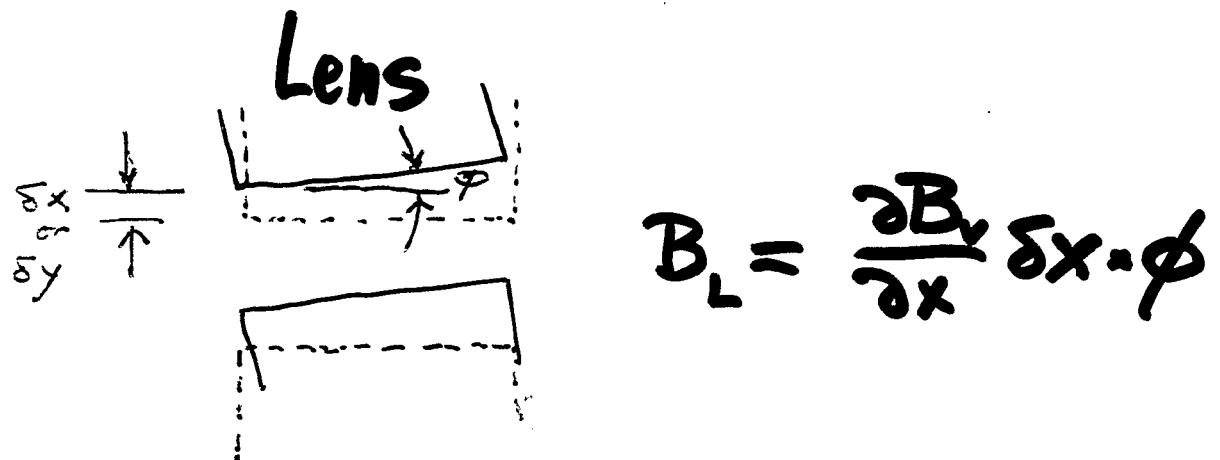
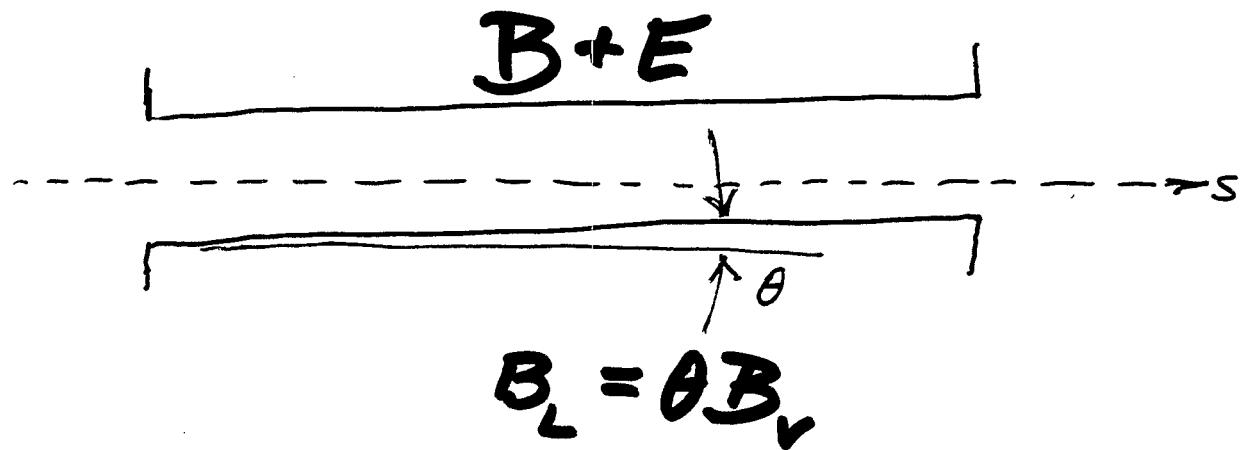
$$\langle B_L(s) \delta\omega_a(s) \rangle$$

1. Where is  $B_L(s)$  from?
2. What is  $\delta\omega_a(s)$ ?
3. Noncommuting rotations  
in this case:  

$$\Phi_L \Phi_R - \Phi_R \Phi_L \rightarrow \Phi_R$$
4. Commuting pairs of  
rotations in this case.
5. Formula. Tolerances.

6a

$$\langle B_L(s) \delta\omega_a(s) \rangle \neq 0$$



Plus in fringe fields

$$\langle B_L(s) \delta\omega_a(s) \rangle \neq 0$$

What is  $\delta\omega_a(s)$ ?

$$\delta\omega_a = -\frac{e}{m} \left[ aB_r - \left( a - \frac{m^2}{p^2} \right) \beta E_R \right] \neq 0$$

1. Inside  $(B+E)$  sections
2. Due to any  $\perp$  shifts  
of lenses (not  
compensated  $\frac{\partial B}{\partial x} \delta x$ )
3. Non-compensated  $\delta\omega_a$ 's  
in fringe fields
4.  $\Delta P/P$
5. Pitch
6. Non-homogeneity  
of  $B$

6c.

$$\langle B_L(s) \delta\omega_a(s) \rangle \neq 0.$$

Along the ideal  
(non-perturbed orbit):

$$\frac{ds_L}{dt} = s_R \delta\omega_0 \cos 2\pi k t / T$$

$$\begin{aligned} \frac{ds_R}{dt} = & -\xi \delta\omega_0 \cos 2\pi k t / T + \\ & + s_r \omega_L \cos(2\pi k t / T + \phi) \end{aligned}$$

$$\frac{ds_r}{dt} = -s_R \omega_L \cos(2\pi k t / T + \phi)$$

$$\omega_L = \frac{e B_{L0}}{m \gamma} (1 + \alpha)$$

$$B_L = B_{L0} \cos(2\pi k s / L + \phi)$$

$$s = vt$$

$$\delta\omega = \delta\omega_0 \cos 2\pi k s / L$$

6d

$$\langle B_L(s) \delta\omega_a(s) \rangle \neq 0$$

Result :

Spin rotation not around L-axis (due to  $B_L$ ), or around V-axis (due to  $\delta\omega_a$ ), but around R-axis!

$$\Omega = \left| \frac{\omega_L \delta\omega_0 \sin\phi}{2k\omega_c} \right|$$

We need

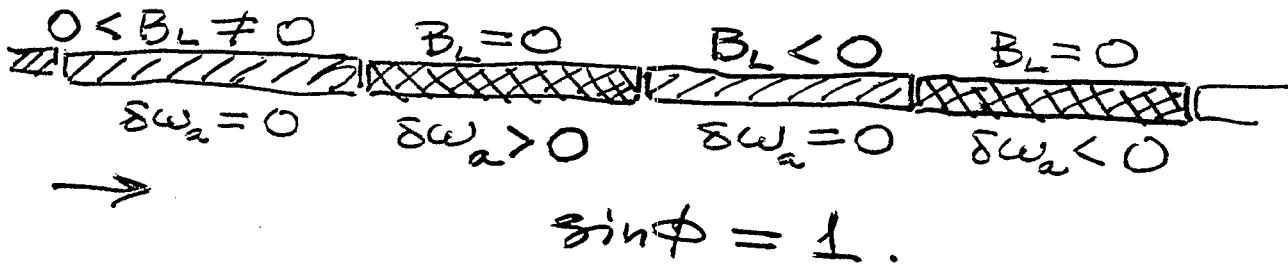
$$\left| \frac{\omega_L \delta\omega_0 \sin\phi}{2k\omega_c} \right| \ll \frac{1}{2} \beta \gamma \omega_c$$

What is going on?  
Why around the R-axis?

6e

$$\langle B_L(s) \delta\omega_a(s) \rangle \neq 0$$

The answer:  
non-commutativity of  
rotations.



$$\sin\phi = 1.$$

$$(\Phi_v^{-1})(\Phi_L^{-1})\Phi_v\Phi_L \underset{\text{state}}{\text{(Spin)}} \neq \underset{\text{state}}{\text{(The same)}}$$

$$[\Phi_v^{-1}\Phi_L^{-1}] = \Phi_L\Phi_v \neq \Phi_v\Phi_L$$

$$\Phi_v\Phi_L \underset{\text{state}}{\text{(Spin)}} \neq \Phi_L\Phi_v \underset{\text{state}}{\text{(The same)}} =$$

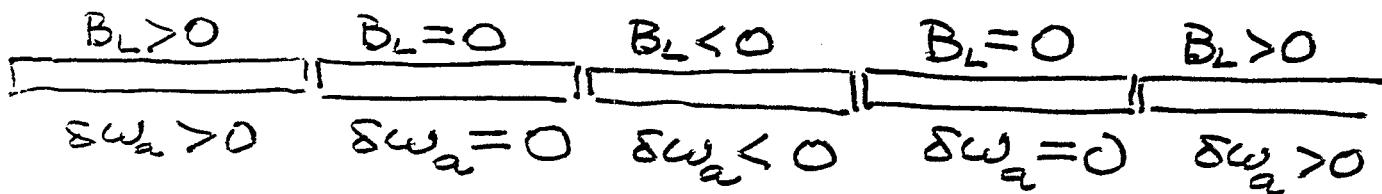
$$= (\Phi_v\Phi_L + \frac{1}{2}\Phi_R) \underset{\text{state}}{\text{(Spin)}}$$

So spin is rotating around  
the R-axis

6f

$$\langle B_L(s) \delta\omega_a(s) \rangle \neq 0$$

Why the case  $\cos\phi = 1$   
 $\sin\phi = 0$   
is different?



Two simultaneous rotation,  
and then inverse of this  
operation — COMMUTE.

$$\langle B_R(s) \delta\omega_a(s) \rangle \neq 0$$

$B_R(s)$  mostly from vertical shifts of lenses,

$$B_r = \left( \frac{\partial B_r}{\partial x} \right) \delta y .$$

Final spin rotation around the longitudinal axis, connecting  $S_r$  with  $S_R$ , not  $S_r$  with  $S_L$ , as in all other cases.

Now  $\boxed{\omega_R = a \frac{e B_{R0}}{m}}$

$$B_r = B_{r0} \cos(2\pi ks/L + \phi)$$

$$\left| \frac{S_R}{S_L} \frac{\omega_{R0} \delta\omega_{ac} \sin\phi}{2k\omega_c^2} \right| \leq \frac{\gamma \beta \gamma}{2}$$

8  
EDM Note No 32  
did not finished

Pitch

I am Sorry

# $\frac{\Delta p}{p}$ : dephasing

To calculate  $\delta\omega_a(s)$  along the horizontally shifted equilibrium orbit due to

$$\Delta p/p,$$

$$\Delta x_{eq}(s) = D(s) \Delta p/p.$$

The average along the orbit,  $\langle \delta\omega_a(s) \rangle \neq 0$  leads to dephasing.

$$\langle \delta\omega_a \rangle = \frac{16}{L} a \frac{eB_v}{m} \left( \frac{\Delta p}{p} + \frac{\Delta P}{P} - \langle \frac{\Delta p}{p} \rangle \right).$$

$$* \left\{ \left[ 2 + \frac{\langle P_{BE} \rangle}{B_v} - \frac{1}{\gamma^2} - 2 \frac{E_R}{B_v} \right] \ell_{BE} + \frac{1}{B_v} \frac{\partial B_v}{\partial x} / \ell_a (D_F - D_B) \right\}$$

(for the LOI design)

$$\langle \delta\omega_a \rangle = a \frac{eB_v}{m} \cdot 2.15 \left[ \frac{\Delta p}{p} \right] + \frac{\Delta P}{P} - \langle \frac{\Delta p}{p} \rangle$$

(a leftover after RF)

9a

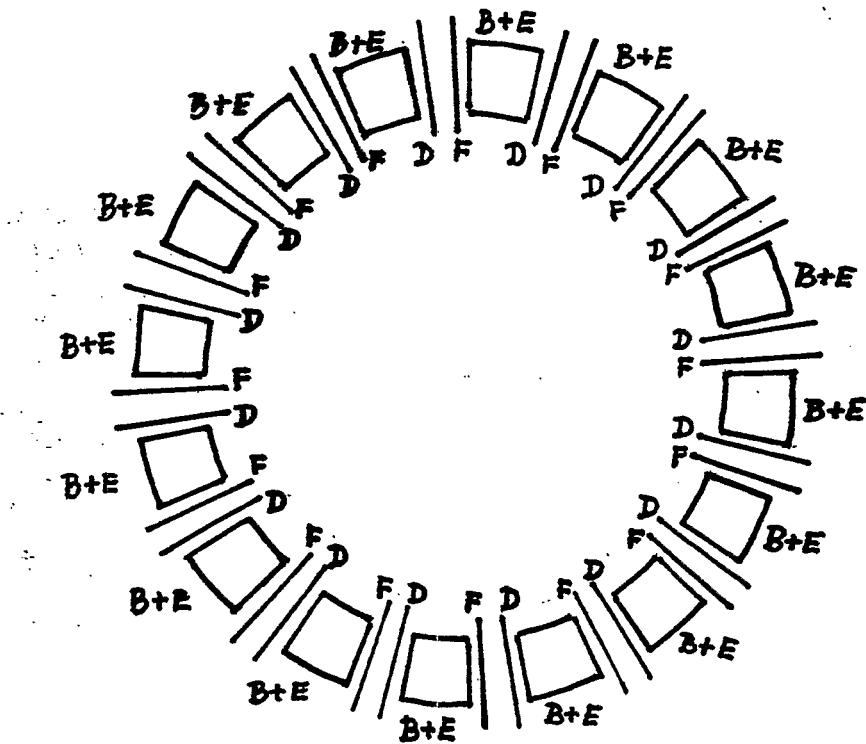


Figure 3: A setup of the EDM ring for the 500 MeV/c case.

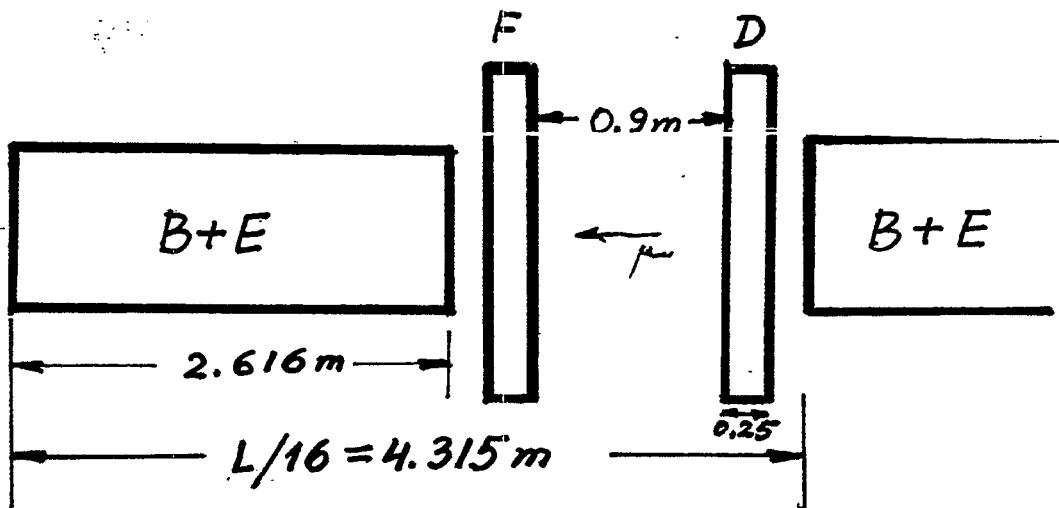
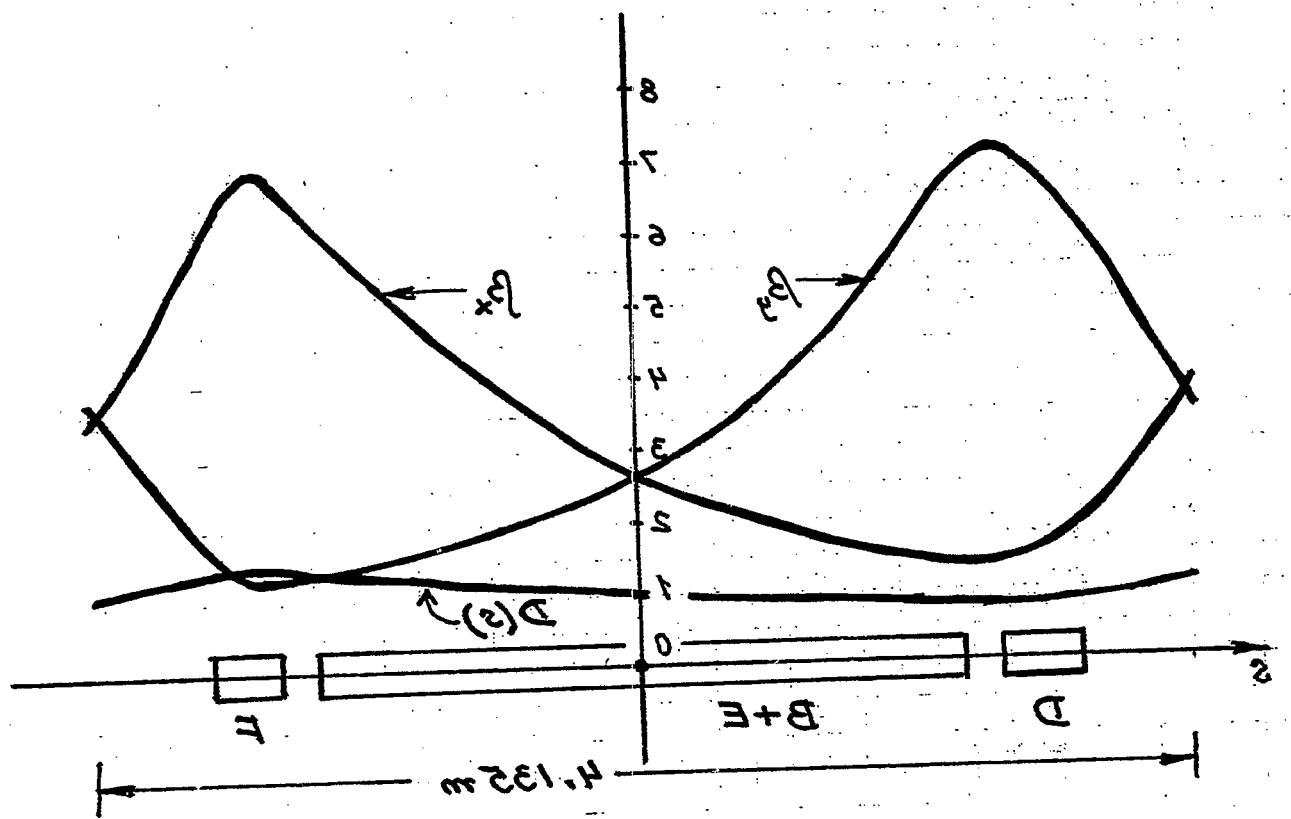


Figure 4 : One period of the EDM ring lattice.  
The ring contains 16 such periods.



Figures  $C$ ,  $(z)B$  and  $(z)D$  are inside one bar of the total area.

$\frac{\Delta P}{P}$  together with  
 $B_L(s)$  &/or  $B_R(s)$

$$\delta\omega_a(s) = -4.36 \alpha \cdot \frac{eB_r}{m} \left\langle \frac{\Delta P}{P} \right\rangle \cos\left(\frac{32\pi s}{L} - 0.33\right)$$

$$B_L(s) = b_L \cos(32\pi s/L + \phi)$$

$$\text{or } B_R(s) = b_R \cos(32\pi s/L + \phi)$$

Use formulas above, pp. 6d, 7

For  $\Delta P/P$  effects,  
quadratic terms  
proportional to  $\langle (\Delta P/P)^2 \rangle$   
must be also  
calculated.

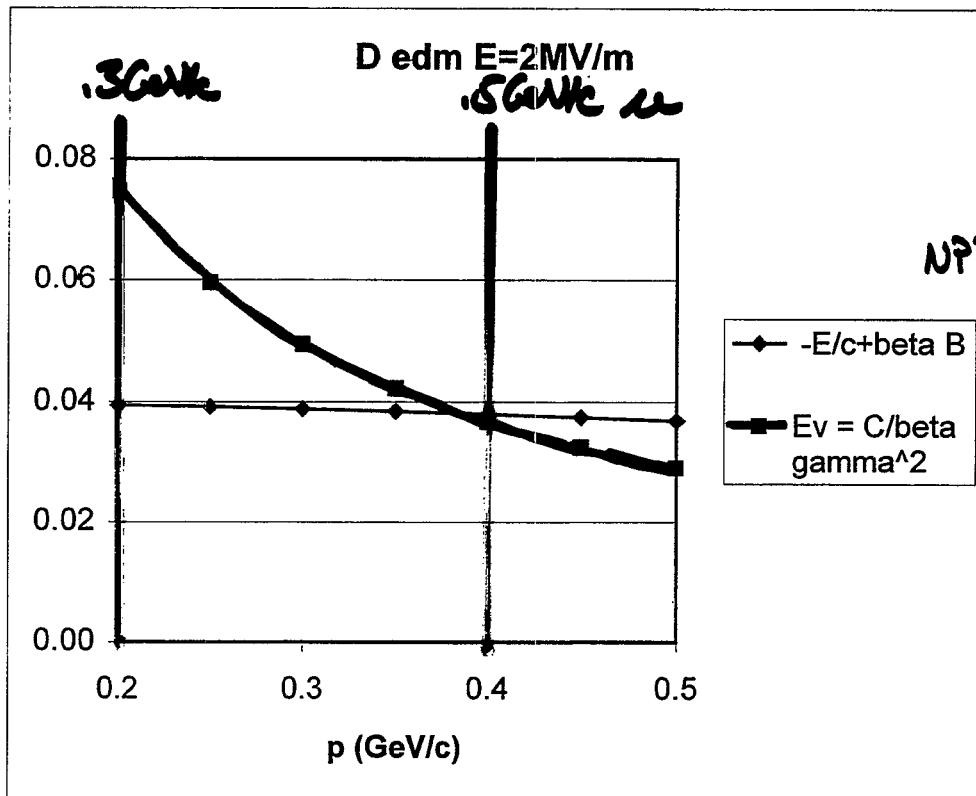
Bill Morse

What is best deuteron p?

$$E = \alpha B c \beta \gamma^2$$

Ring parameters as a function of deuteron momentum for a circular ring.

MV/m	p(GeV/c)	$\beta = pc/E$	$\gamma = E/m$	B (T)	R (m)	$-E/c + \beta \times B$	$E_v = C/\beta\gamma^2$
2	0.1	0.053	1.001	0.874	0.45	0.040	0.187
2	0.15	0.080	1.003	0.582	1.00	0.040	0.125
2	0.2	0.106	1.006	0.435	1.79	0.039	0.093
2	0.25	0.132	1.009	0.347	2.81	0.039	0.074
2	0.3	0.158	1.013	0.288	4.07	0.039	0.062
2	0.35	0.183	1.017	0.246	5.57	0.038	0.053
2	0.4	0.208	1.022	0.214	7.33	0.038	0.046
2	0.45	0.233	1.028	0.189	9.35	0.037	0.041
2	0.5	0.257	1.035	0.169	11.64	0.037	0.036



$$\vartheta_0 = \frac{13.6 MeV}{\beta c p} \sqrt{x/X_0} [1 + 0.038 \ln(x/X_0)]$$

$$\varepsilon_b = \varepsilon_s / \beta \gamma$$

$$p = 0.25644c \quad \vartheta_0 = 412 \text{ mm} \sqrt{x/X_0}$$